A

20. A particle of mass m and energy E, moving in the positive x-direction, encounters a one-dimensional potential barrier at $x = 0$. The barrier is defined by

$$V = 0 \text{ for } x < 0$$

$$V = V_0 \text{ for } x \geq 0 \ (V_0 \text{ is positive and } E > V_0)$$

If the wave function of the particle in the region $x < 0$ is given as $A\,e^{ikx} + B\,e^{-ikx}$,

(a) Find the ratio $\dfrac{B}{A}$.

(b) If $\dfrac{B}{A} = 0.4$, find $\dfrac{E}{V_0}$, and the transmission and reflection coefficients.

21. (a) Establish the equation $dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left[T\left(\frac{\partial P}{\partial T}\right)_V - P\right]dV$, given that $dU = TdS - PdV$ and $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$, where U, P, T, V and S are, respectively, the internal energy, pressure, temperature, volume and entropy of the system.

(b) If the specific heat is taken to be independent of T, utilize the above equation to derive an expression for $U(T, V)$ for one mole of a van der Waals gas and then obtain the corresponding expression for an ideal gas.

22. Solve the differential equation $xy \frac{dy}{dx} = 3y^2 + x^2$ with the initial condition $y = 2$ when $x = 1$.